

# Calculating Confidence Intervals for the Mean of a Lognormally Distributed Variable

T. B. Parkin,\* S. T. Chester, and J. A. Robinson

## ABSTRACT

Many physical, chemical, and biological properties of soils exhibit skewed distributions that can be approximated by the two-parameter lognormal distribution. Recent attention in the soils literature has focused on the best method of estimating the mean for lognormally distributed variables; however, little attention has been given to methods for constructing confidence intervals about the mean of such variables. This paper reports on the efficacy of five different methods, one exact and four approximate, for constructing confidence intervals about the mean of a lognormally distributed variable. These methods were examined over a range of sample sizes ( $n = 6-100$ ) by using Monte Carlo simulation to draw samples from four lognormal distributions with known mean and variances (coefficients of variation of 50, 100, 200, and 500%). Performance of the confidence-interval methods were assessed by identifying how close the calculated probability levels for the upper and lower confidence limits came to the stated probability levels. Two commonly applied methods, the asymptotic and Patterson's methods, failed to provide accurate probabilistic coverage. A nonparametric method developed by the authors was superior to both of these methods and compared favorably with Land's method in some cases. The latter is an exact method that gives the correct probabilistic coverage. The third method, attributed to Cox, was generally inferior to Land's and our nonparametric method, although it outperformed the asymptotic and Patterson's methods. For precise work, Land's method should be employed for constructing confidence intervals about the mean of a lognormally distributed variable.

**I**N QUANTITATIVE SOILS STUDIES where a probabilistic justification is needed for the inferences drawn from sample data, it is essential that measures of variability or confidence-interval estimates be reported along with parameter estimates. Reporting only point estimates of parameters, regardless of the nature of the variables measured, should not be considered sufficient (Robinson, 1985). When data are normally distributed, estimation of confidence intervals about the mean is straightforward (e.g., Snedecor and Cochran, 1967; Steel and Torrie, 1980). However, many of the

physical, chemical, and biological properties of soils display skewed distributions, which are better approximated by the two-parameter lognormal distribution (Parkin et al., 1988; Warrick et al., 1977). The confidence-interval estimation procedure applicable for Gaussian-distributed data is suboptimal when applied to lognormal data, and the situation worsens as the degree of skewness or variance increases.

There have been many methods proposed for calculating approximate confidence limits about the mean of a lognormal variable (Mood, 1950; Aitchinson and Brown, 1957; Patterson, 1966; Sichel, 1966; Blais and Carlier, 1968; Hoyle, 1968; Sokal and Rohlf, 1969; Land, 1971, 1972). All of these methods differ with regard to the calculated upper and lower confidence limits for a given data set, yet few recommendations exist regarding the application and implementation of these methods. Koch and Link (1970) describe five methods of confidence-interval calculation used in the geological sciences and illustrate their use with a sample data set. However, a detailed evaluation to determine the optimum method was not performed.

A method for the calculation of nearly exact confidence intervals was developed by Sichel (1966). This method is based on the distribution function of a test statistic and requires the use of tabulated constants. Land (1971) developed a method for computing confidence intervals for linear functions of the normal distribution, and applied this technique to obtain exact confidence intervals for the lognormal distribution. In a subsequent study, he compared the efficacy of this technique to four other approximate methods (Land, 1972). This evaluation was performed over a wide range of population variances, but only for sample sizes of  $n = 11, 101$ , and  $1001$ .

Despite the fact that soil variables have often been reported to be lognormally distributed, there have been few applications of these confidence-interval estimation techniques to soils data. Notable exceptions are studies of soil denitrification (Folorunso and Rolston, 1984; Parkin et al., 1985, 1987; Parkin and Robinson, 1989). The paucity of examples in the soils literature where these methods have been applied belies the importance of the use of lognormal confidence-interval estimation procedures. Methods exist

USDA-ARS, Environmental Chemistry Lab., Bldg. 008, Room 102, BARC-West, Beltsville, MD 20705. Contribution from the USDA-ARS, Environmental Chemistry Lab. and The Upjohn Co., Kalamazoo, MI. Received 27 Jan. 1989. \*Corresponding author.

that are substantially better than those that have been previously applied or recommended in common statistical text books.

In a previous study (Parkin et al., 1988), we reported on three methods for estimating the mean, variance, and coefficient of variation for lognormally distributed variables. The present study extends these findings by evaluating several methods of computing confidence intervals for a lognormal mean. The procedures we chose to evaluate include those that Land (1972) reported work well, and an approximate approach developed by the authors and based on quantiles. These methods are evaluated over a range of distributions and sample sizes representative of those commonly observed in studies of soil variables.

## METHODS

### *Confidence-Interval Estimation Procedures*

In this study, five methods for constructing a confidence interval for the mean of a lognormally distributed variable were evaluated. Three of the methods have been applied to soils data, while the other two have not. It should be noted that we are explicitly interested in constructing a confidence interval for the mean of the lognormal distribution and not the median. The equations for the mean and median for the two-parameter lognormal distribution are

$$\begin{aligned}\text{Median} &= \exp(\mu) \\ \text{Mean} &= \exp(\mu + \sigma^2/2)\end{aligned}$$

where  $\mu$  and  $\sigma^2$  are the mean and variance of the distribution of logarithms of the lognormal random variables. Thus, confidence intervals for the mean of the logarithms of the observations (Sokal and Rohlf, 1969; Green 1979) are associated with the median and not the mean. A brief description of the five methods evaluated in this study are described below. Mathematical details on the implementation of these methods is provided in the Appendix.

Method 1 is the asymptotic or normal theory method where the mean and standard error are estimated from sample data. This method is recommended in all introductory statistics textbooks when one has Gaussian data (e.g., Snedecor and Cochran, 1967; Steel and Torrie, 1980). This method is appropriate for lognormal data when either the skewness is negligible or the sample size is large enough for the Central Limit Theorem to apply. It is not appropriate for lognormal distributions having high variance when small sample sizes are used. This method is included in our evaluations to demonstrate how poorly it performs in such situations.

Method 2 was suggested by Patterson (1966). This method has been applied by soil scientists in investigations of soil denitrification (Folorunso and Rolston, 1984; Parkin et al., 1985), and was one of the methods investigated by Land (1972).

Method 3 was suggested by D.R. Cox (Land, 1972). The derivation of this method is based on constructing a confidence interval about  $(\mu + \sigma^2/2)$  using the first two moments of the sample mean and variance of the logarithms of the original observations, and then exponentiating the results. In his study, Land (1972) found that it outperformed all other approximate methods.

Method 4 is a nonparametric quantile method that uses the order statistics of the untransformed observations. The quantile corresponding to the mean of the lognormal distribution is estimated and confidence intervals for this quantile are constructed (Guenther, 1973). This method was de-

veloped by the authors as a robust alternative to the other four parametric methods evaluated in this study.

Method 5 is the exact method of Land (1971). Land's method is the benchmark because it provides exact confidence limits. Thus, the realized Type I error rate exactly equals the nominally stated level. The simulation results for Land's methods are reported to evaluate the precision of our simulation algorithms only. Since Land's method is exact, it is unnecessary to evaluate it using Monte Carlo simulation.

### *Confidence-Interval Evaluations*

The five confidence-interval methods were examined using Monte Carlo simulation. Four lognormal distributions that are representative of the positively skewed distributions observed for soils data (Parkin et al., 1988) were used in the simulations. Each population had a mean of 10 and coefficients of variation of 50, 100, 200, and 500%. The statistical properties of these two-parameter lognormal populations are given in Table 1. Three of these distributions were used in our previous study (Parkin et al., 1988) to evaluate three methods for estimating the mean, variance, and coefficient of variation of lognormally distributed data. An additional population (CV = 500%) is used in this study.

For each lognormal population (Table 1), the five confidence-interval estimation procedures were evaluated for sample sizes from 6 to 100 in steps of 2. A total of 25 000 simulations were performed at each sample size for each lognormal population. Computer programs in both BASIC and FORTRAN were written to draw variates from lognormal distributions and to calculate the confidence intervals for the five methods investigated. For the purposes of this study, an  $\alpha$  level of 10% was chosen with the error divided equally between the two tails. This is equivalent to constructing two one-sided 95% confidence limits, thus yielding a two-sided 90% confidence interval.

One way of defining optimality for two-sided confidence intervals is to find the narrowest interval among all intervals with the same probability of coverage. While the equal division between the two tails provides the shortest intervals for symmetric distributions, it does not when the distribution is skewed. However, since our interest is comparison of the methods, the fact that equal division does not result in computation of the shortest interval is of no relevance to this study.

For each confidence-interval estimation procedure, the proportion of times the actual population means was greater than the upper confidence limit was counted (5% expected for  $\alpha = 0.10$ ). Likewise, the proportion of times the population mean was less than the lower limit was counted (5% expected for  $\alpha = 0.10$ ). The results of this analysis are estimates of the actual probability limits for each method. Thus, the methods are judged by how well these estimated probability limits compare with the theoretical 5% target level. Simulations were also performed at  $\alpha$  levels of 0.05 and 0.01 for selected sample sizes to ascertain the consistency of our results.

The average upper and lower confidence limits of the 25 000 simulations runs at each sample size were also computed to enable calculation of the average width for a given confidence-interval estimation procedure.

**Table 1. Statistical properties of the four test populations used for evaluation.**

Popula- tion	Mean	Median	Mode	Skewness	Vari- ance	Mean of logs	SD of logs
A	10.0	8.944	7.155	1.625	25.0	2.191	0.4734
B	10.0	7.071	3.536	4.000	100.0	1.956	0.8326
C	10.0	4.472	0.894	14.000	400.0	1.498	1.2690
D	10.0	1.961	0.075	140.000	2500.0	0.674	1.8050

## RESULTS AND DISCUSSION

The performance of the different methods in calculating the lower confidence limit is presented in Fig. 1, which shows the actual probability levels as a function of sample size. With samples drawn from Population A (Fig. 1A) the asymptotic method, Cox's method, and our quantile method (Methods 1, 3, and 4, respectively) yield limits that are slightly less than the theoretical level of 0.05, while the limits provided by Patterson's method and Land's method (Methods 2 and 5) are essentially equal to 0.05. As population skewness increases, differences between the calculated confidence levels associated with each method increase. Thus, for Population D (Fig. 1D) Patterson's and the quantile methods overestimate the theoretical probability level of 0.05, while the asymptotic and Cox's methods underestimate it. Land's method provides an exact 0.05 limit for an  $\alpha$  level of 0.10. Our results yielded confidence limits that ranged from 0.047 to 0.051 over the range of sample sizes for each population, indicating that the simulation error was well below the differences observed between the methods.

Upper confidence limits for samples drawn from the test populations are presented in Fig. 2. For Population A (Fig. 2A), the quantile method yields confidence limits that are slightly less than the 0.05 level, and Patterson's, Cox's, and the asymptotic methods

yield estimates that are slightly greater than 0.05. For the highly skewed populations (Fig. 2C and 2D), the confidence limits provided by all the approximate methods exceeded the theoretical 0.05 level. The upper  $P$  values calculated by Patterson's, Cox's, and the asymptotic methods are particularly poor at low sample sizes ( $n < 12$ ). As sample size increases, however, Cox's and the quantile methods provide confidence limits that approach the 0.05 level. Results of the method evaluations at two-sided  $\alpha$  levels of 0.05 and 0.01 revealed the same pattern as observed in Fig. 1 and 2 (data not shown).

These results indicate that the asymptotic method and Patterson's method, which have often been applied to soils data in the past, are inferior to the other approximate methods (Cox's and the quantile method). For all four populations, Land's method provides exact upper and lower confidence limits of 0.05, with the resulting confidence-interval coverage of 90%.

An additional approximate method, derived by Sichel (1966), was also investigated. While this nearly exact method did not compare with Land's method for coverage, it actually outperformed the other approximate methods for all populations. While the other approximate methods could be calculated with readily available statistics tables, Sichel's method requires the same kind of specialized tables used by Land's method. We do not present results of Sichel's

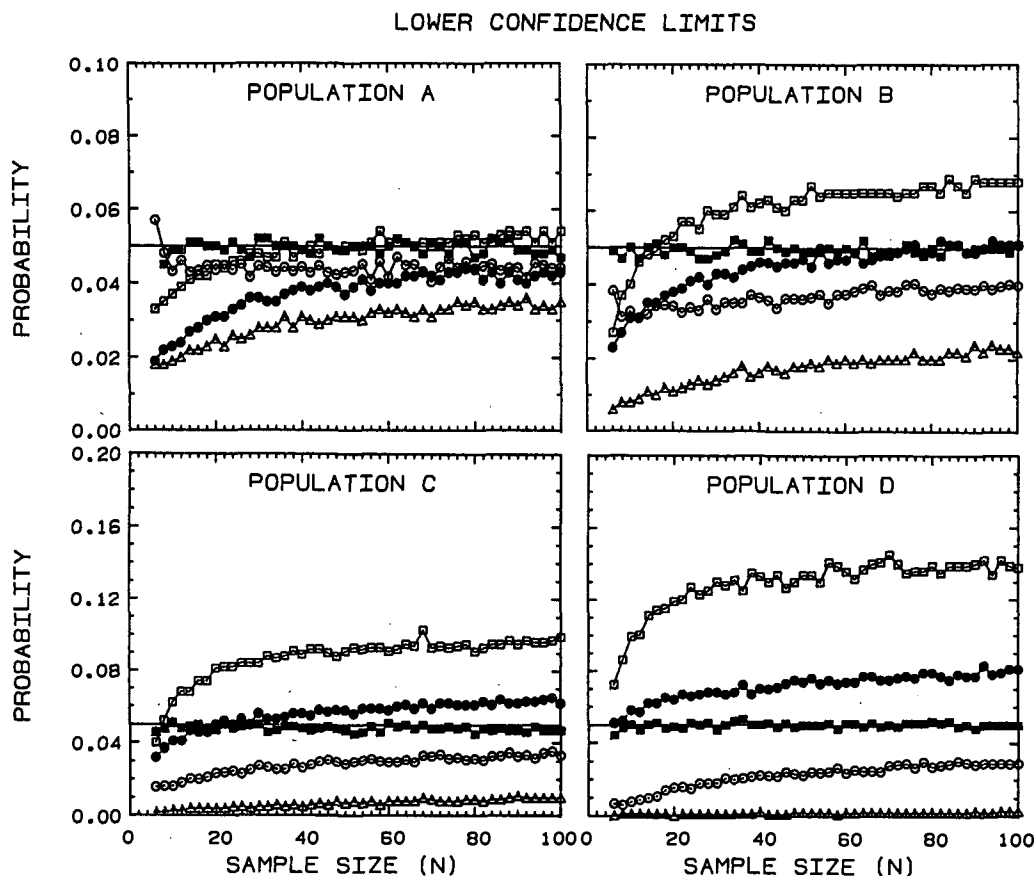


Fig. 1. Realized probability level ( $\alpha/2$ ) of lower confidence limits calculated by the five methods for each of the four lognormal test populations. Symbols: asymptotic ( $\Delta$ ), Patterson's ( $\square$ ), Cox's ( $\circ$ ), quantile ( $\bullet$ ), and Land's ( $\blacksquare$ ) methods. The horizontal line in each panel indicates the theoretical 0.05 probability level. Note differences in vertical scales of panels A-B and C-D.

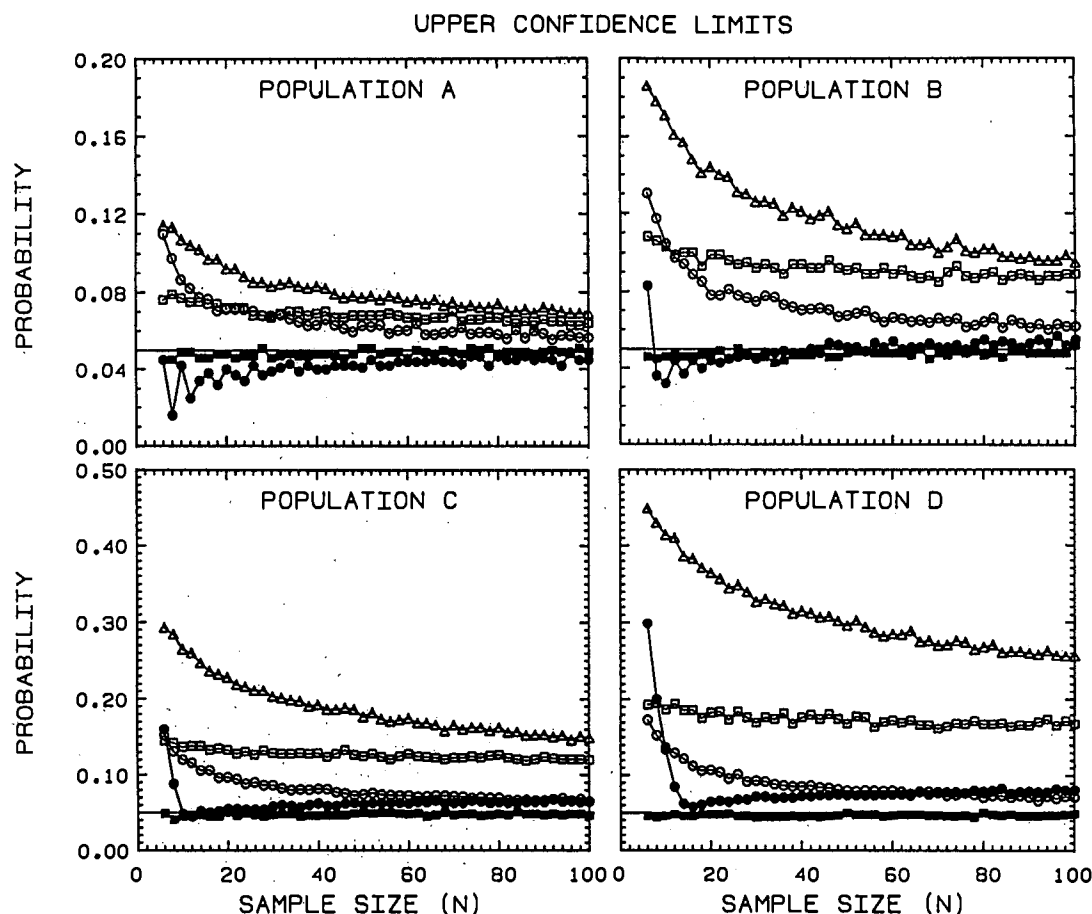


Fig. 2. Realized probability level ( $\alpha/2$ ) of upper confidence limits calculated by the five methods for each of the four lognormal test populations. Symbols: asymptotic ( $\Delta$ ), Patterson's ( $\square$ ), Cox's ( $\circ$ ), quantile ( $\bullet$ ), and Land's ( $\blacksquare$ ) methods. The horizontal line in each panel indicates the theoretical 0.05 probability level. Note differences in vertical scales of panels A-B and C-D.

method here, since it offers no computational advantage over Land's method.

One consequence of sampling from highly skewed lognormal populations is that, with low sample sizes, the resulting confidence intervals are very large. Confidence-interval widths for samples drawn from Population D are presented in Table 2. It is observed that, for small  $n$ , the extreme widths of the confidence intervals make them of little use in discerning treatment effects. The lack of power inherent in comparisons of samples from highly skewed populations would not justify conducting the experiment if an  $\alpha$  level of 0.1 is required.

Based on the simulation results, we have developed

Table 2. Average widths of confidence intervals calculated by the methods for selected sample sizes from Population D.

Sample size	Interval width
$n$	
8	$2.0 \times 10^7$
12	2110
16	237
20	98.0
30	42.4
40	28.4
60	18.2
100	11.9

Values reported are the average widths of 25 000 simulations performed at each sample size.

some recommendations regarding the application of these confidence-interval estimation procedures (Table 3). These recommendations are provided for three ranges of sample sizes drawn from low to moderately skewed populations (CVs 50–100%) and drawn from highly skewed populations (CVs 200–500%).

For all the lognormal populations evaluated, Land's method is the preferred method, as it provides exact coverage at the stated probability level. None of the approximate methods work well for small sample sizes ( $n < 20$ ); however, with  $n > 20$ , our quantile method provided reasonably accurate coverage. A recommendation is given for this method because it is simple to implement, and is based on techniques found in elementary statistics texts. With sample sizes

Table 3. Practical recommendations for methods of calculating 90% confidence intervals about the mean of a lognormal variable.

Sample size	Population coefficient of variation	
	50–100%	200–500%
	method†	
6–12	5	5
14–40	5,4	5,4
40–100	5,4,3	5,3,4

† Method 5 = Land's (1971) method, Method 4 = the nonparametric quantile method, and Method 3 = Patterson's (1966) method. Multiple recommendations are listed in order of decreasing preference.

in the range of 60 to 100, Cox's approximate method also provides nearly exact coverage. With highly skewed populations, Cox's method is superior to our quantile method. This method is also simple to implement, requiring only a table of  $t$  values.

Regarding the other methods, the asymptotic method should only be applied when skewness is negligible. While the Central Limit Theorem states that, for a large-enough sample size, this method will be accurate, it is clear from the simulation results that even a sample size of 100 is not large enough for the theorem to be applicable. Patterson's method also cannot be recommended, since Cox's method is only slightly harder to compute and clearly outperforms it.

## CONCLUSIONS

From the results of this study, several specific conclusions are drawn:

1. Of the methods recommended in the literature, the asymptotic and Patterson's methods have been the ones most often applied to soils data, yet these methods perform poorly with regard to providing coverage at the stated probability level.
2. Land's method is the preferred method for all situations, as it provides exact confidence limits.
3. With large sample sizes ( $n > 60$ ), Cox's method is a suitable alternative, since it provides reasonably accurate coverage and it is simple to implement.
4. The quantile method we developed has applications for medium to large sample sizes ( $n = 40$ – $100$ ). This method was developed as a robust alternative to the exact method. Since, in practice, it is impossible to deduce the true underlying population from sample data, this method may have utility when the underlying population deviates from true lognormality.

Finally, the purpose of this study was to evaluate confidence-interval methods applied to lognormal data. While the lognormal distribution may be an appropriate model for many soil processes (Parkin et al., 1988), there are situations in which the data, though skewed, are not lognormally distributed. The efficacy of these confidence-interval methods in providing accurate coverages in situations when the underlying population cannot be adequately modeled by the lognormal distribution is currently under evaluation.

## ACKNOWLEDGMENTS

The authors wish to thank Bruce Tesar for writing computer programs used in a portion of the Monte Carlo simulations.

## APPENDIX

Details on the implementation of the five confidence-interval estimation procedures are presented here. The following terms are used in the implementation of the methods.

- $\bar{x}$  = the arithmetic average of the sample values.
- $s^2$  = the sample variance of the values (unbiased form).
- $\bar{\mu}$  = the arithmetic average of the log-transformed sample values (natural logarithms).

$\hat{\sigma}^2$  = the sample variance of the log-transformed values (unbiased form).

$t$  = the critical value from the Student's  $t$  distribution with  $n - 1$  degrees of freedom for a two-sided  $\alpha$  value of  $\alpha = 0.10$ .

$z_{1-\alpha/2}$  = the ordinate from the standard Gaussian distribution [ $N(0,1)$ ], (1.645 for  $\alpha = 0.10$ ).

$\hat{p}$  = an estimate of the quantile probability associated with the mean of a two-parameter lognormal distribution.

$\Phi$  = the standard Gaussian cumulative distribution.

$C_L(v, \hat{\sigma}, \alpha/2)$  = the constant from Land's (1971) table of lower confidence-limit factors for  $v$  degrees of freedom, and an  $\alpha$  level of 0.10.

$C_U(v, \hat{\sigma}, \alpha/2)$  = the constant from Land's (1971) table of upper confidence-limit factors for  $v$  degrees of freedom, and an  $\alpha$  level of 0.10.

$LCL_m$  = the lower confidence limit calculated by method  $m$ .

$UCL_m$  = the upper confidence limit calculated by method  $m$ .

*Method 1.* This is the asymptotic method, which is typically applied when the population is assumed to be normally distributed (Steel and Torrie, 1980). Lower and upper confidence limits are given by

$$LCL_1 = \bar{x} - t \sqrt{s^2/n} \quad [1]$$

$$UCL_1 = \bar{x} + t \sqrt{s^2/n} \quad [2]$$

*Method 2.* This is the method recommended by Patterson (1966). Lower and upper confidence limits are calculated by

$$LCL_2 = \exp(\bar{\mu} + \hat{\sigma}^2/2 - t\sqrt{\hat{\sigma}^2/n}) \quad [3]$$

$$UCL_2 = \exp(\bar{\mu} + \hat{\sigma}^2/2 + t\sqrt{\hat{\sigma}^2/n}) \quad [4]$$

*Method 3.* This is the method devised by Cox as described by Land (1972). Lower and upper confidence limits are calculated by

$$LCL_3 = \exp\left(\bar{\mu} + \hat{\sigma}^2/2 - t(\hat{\sigma}/\sqrt{n})\sqrt{1 + \frac{\hat{\sigma}^2 n}{2(n+1)}}\right) \quad [5]$$

$$UCL_3 = \exp\left(\bar{\mu} + \hat{\sigma}^2/2 + t(\hat{\sigma}/\sqrt{n})\sqrt{1 + \frac{\hat{\sigma}^2 n}{2(n+1)}}\right) \quad [6]$$

*Method 4.* This method was developed by the authors and is based on the quantile corresponding to the mean of the lognormal distribution ( $p$ ), which is defined by the equation

$$p = \Pr[x \leq E(X)] = \Phi(\sigma/2) \quad [7]$$

where  $X$  is distributed as a lognormal random variable.

If we estimate  $\sigma$  by  $\hat{\sigma}$  (the positive square root of the variance of the log-transformed values), then an estimate of  $p$ ,  $\hat{p}$ , can be computed by solving Eq. [7] using  $\hat{\sigma}$ . Thus, non-parametric methods can be used to construct a confidence interval about  $p$  (Guenther, 1973). The confidence limits are estimated by the selection of the appropriate order statistics from the sample

$$LCL_4 = x(r) \quad [8]$$

$$UCL_4 = x(s) \quad [9]$$

where  $x(r)$  and  $x(s)$  are the  $r$ th and  $s$ th order statistics ( $r < s$ ). Explicitly,  $r$  is defined to be the smallest integer such that Eq. [10] is achieved, while  $s$  is the largest integer such that Eq. [11] is achieved.

$$\sum_{i=0}^{r-1} \binom{n}{i} \hat{p}^i (1 - \hat{p})^{n-i} \geq 0.05 \quad [10]$$

$$\sum_{i=0}^{s-1} \binom{n}{i} \hat{p}^i (1 - \hat{p})^{n-i} \leq 0.95 \quad [11]$$

For sample sizes large enough, ( $n > 20$ ), the following formulas may be used:

$$r = n\hat{p} - z_{0.95} \sqrt{n\hat{p}(1 - \hat{p})} \quad [12]$$

$$s = n\hat{p} + z_{0.95} \sqrt{n\hat{p}(1 - \hat{p})} \quad [13]$$

Since Eq. [12] and [13] will rarely give integer solutions,  $r$  and  $s$  are determined by rounding the results to the next highest integer value.

**Method 5.** This is the exact method developed by Land (1971). Lower and upper confidence limits are given by

$$\text{LCL}_5 = \exp(\bar{\mu} + \hat{\sigma}^2/2 + \hat{\sigma} C_L / \sqrt{(n-1)}) \quad [14]$$

$$\text{UCL}_5 = \exp(\bar{\mu} + \hat{\sigma}^2/2 + \hat{\sigma} C_U / \sqrt{(n-1)}) \quad [15]$$

where  $C_L$  and  $C_U$  are factors calculated from a function that depends on the number of observations ( $n$ ), the standard deviation of the log-transformed values ( $\sigma$ ) and the  $\alpha$  level selected. This function was developed and evaluated by Land (1971, 1975), and a computer algorithm for computing these factors has been developed (Land, 1987). Linear interpolation of Land's (1971) tables of lower and upper confidence-limit factors were used in the evaluations of this work.

The BASIC programs for the calculation of confidence intervals using each of the above methods are available from the authors.

## REFERENCES

- Aitchinson, J., and J.A.C. Brown 1957. The lognormal distribution. Cambridge Univ. Press, London.
- Blais, R.A., and P.A. Carlier. 1968. Applications of geostatistics in ore evaluation, p. 48-61. In *Ore reserve estimation and grade control*. Spec. Vol. 9. Can. Inst. Min. Metall., Montreal.
- Folorunso, O.A., and D.E. Rolston. 1984. Spatial variability of field-measured denitrification gas flux. *Soil Sci. Soc. Am. J.* 48:1214-1219.
- Green, R.H. 1979. Sampling design and statistical methods for environmental biologists. John Wiley & Sons, New York.
- Guenther, W.C. 1973. Concepts of statistical inference. 2nd ed. McGraw-Hill, New York.
- Hoyle, M.H. 1968. The estimation of variables after using a Gaussianizing transformation. *Ann. Math. Stat.* 39:1125-1143.
- Koch, G.S., and R.F. Link. 1970. Statistical analysis of geological data. John Wiley & Sons, Inc., New York.
- Land, C.E. 1971. Confidence intervals for linear functions of the normal mean and variance. *Ann. Math. Stat.* 42:1187-1205.
- Land, C.E. 1972. An evaluation of approximate confidence interval estimation methods for lognormal means. *Technometrics* 14:145-158.
- Land, C.E. 1975. Tables of confidence limits for linear functions of the normal mean and variance. p. 385-419. In *Selected tables in mathematical statistics*. Vol. III. Am. Math. Soc., Providence, RI.
- Land, C.E. 1987. Hypothesis tests and interval estimates. In E.L. Crow and K. Shimizu (ed.) *Lognormal distributions theory and applications*. Marcel Dekker, Inc., New York.
- Mood, A.M. 1950. Introduction to the theory of statistics. McGraw-Hill, New York.
- Parkin, T.B., J.L. Starr, and J.M. Meisinger. 1987. Influence of sample size on measurement of soil denitrification. *Soil Sci. Soc. Am. J.* 51:1492-1501.
- Parkin, T.B., A.J. Sexstone, and J.M. Tiedje. 1985. Comparison of field denitrification rates determined by acetylene-based soil core and nitrogen-15 methods. *Soil Sci. Soc. Am. J.* 49:94-99.
- Parkin, T.B., J.J. Meisinger, S.S. Chester, J.L. Starr, and J.A. Robinson. 1988. Evaluation of statistical estimation methods for log-normally distributed variables. *Soil Sci. Soc. Am. J.* 52:323-329.
- Parkin, T.B. and J.A. Robinson. 1989. Stochastic models of soil denitrification. *Appl. Environ. Microbiol.* 55:72-77.
- Patterson, R.L. 1966. Difficulties involved in the estimation of a population using transformed sample data. *Technometrics* 8:535-537.
- Robinson, J.A. 1985. Determining microbial kinetic parameters using nonlinear regression analysis: Advantages and limitations in microbial ecology. *Adv. Microb. Ecol.* 8:61-114.
- Sichel, H.S. 1966. The estimation of means and associated confidence limits for small samples from lognormal populations. p. 106-122. In *Symp. Math. Stat. Computer Appl.*, Johannesburg, S. Africa. S. Afr. Inst. Min. Metall., Johannesburg.
- Snedecor, G.W., and W.G. Cochran. 1967. Statistical methods. 6th ed. Iowa State Univ., Ames, IA.
- Sokal, R.R., and F.J. Rohlf. 1969. Biometry. W.H. Freeman & Co., New York.
- Steel, R.G.D., and J.H. Torrie. 1980 Principles and procedures of statistics: A biometric approach. McGraw-Hill Book Co., New York.
- Warrick, A.W., G.J. Mullen, and D.R. Nielsen. 1977. Scaling field measured soil hydraulic properties using a similar media concept. *Water Resour. Res.* 13:355-362.